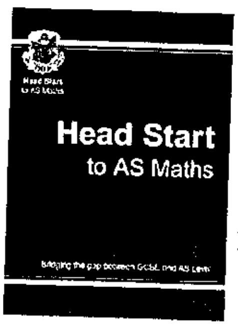
LCHS AS Level Mathematics Induction Booklet

By the time you start AS Level lessons here it may be nearly three months since you took your GCSE examinations and your Mathematics may be a little rusty. This booklet contains a small sample of questions on fundamental GCSE topics which are essential for a successful start to AS Mathematics. It is <u>vitally important</u> that you spend some time working through the questions in this booklet over the Summer as you will need to have a good knowledge of these topics <u>before</u> you commence the course in September. You should have met all the topics before at GCSE. Work through the examples for each topic making sure that you understand them. Then tackle the exercise below, not necessarily every question but enough to ensure you understand the topic thoroughly. The questions increase in difficulty and the answers are given at the back of the booklet.

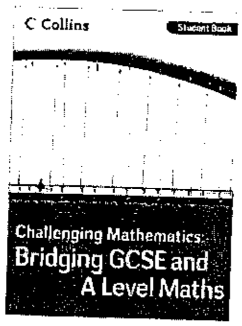
You may also find one of the following books useful which you can purchase new or used from Amazon or eBay.



Head Start to AS Maths

CGP Books

ISBN 1841469939



Collins Maths - Bridging GCSE and A Level: Student Book

Mark Rowland

ISBN 0007410239

Using this booklet and one of the above books will ensure you have a confident start to your AS Level work.

Enjoy the Summer break and we look forward to meeting you after the holidays,

The LCHS Mathematics Department

Expanding and Simplifying Simple Algebraic Expressions

To remove a single bracket, multiply every term in the bracket by the number or the expression on the outside.

$$-2(2x - 3) = (-2)(2x) + (-2)(-3)$$
$$= -4x + 6$$

1) Multiply out the following brackets and simplify.

a) $-3(5x - 7)$ b) $5a - 4(3a - 1)$ c) $5(2x - 1) - (3x - 4)$

To expand two brackets, we must multiply everything in the first bracket by everything in the second bracket. We can do this in a variety of ways, including the smiley face method, FOIL (Fronts Outers Inners Lasts) or using a grid.

$$(x - 2)(2x + 3) = x(2x + 3) - 2(2x + 3)$$
$$= 2x^2 + 3x - 4x - 6$$
$$= 2x^2 - x - 6$$

or

$$(x - 2)(2x + 3) = 2x^2 - 6 + 3x - 4x = 2x^2 - x - 6$$

or

	x	-2
$2x$	$2x^2$	$-4x$
3	$3x$	-6

$$(2x + 3)(x - 2) = 2x^2 + 3x - 4x - 6$$
$$= 2x^2 - x - 6$$

2) Expand and simplify.

a) $(2x + 3y)(3x - 4y)$ b) $4(x - 2)(x + 3)$ c) $(3 + 5x)(4 - x)$

Special Cases

Perfect Square:

$(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$

$(2x - 3)^2 = (2x - 3)(2x - 3) = 4x^2 - 12x + 9$

Difference of two squares:

$(x - a)(x + a) = x^2 - a^2$

$(x - 3)(x + 3) = x^2 - 3^2 = x^2 - 9$

3) Expand and simplify.

a) $(3x + 5)^2$ b) $(7x - 2)^2$ c) $(3x + 1)(3x - 1)$ d) $(5y - 3)(5y + 3)$

Factorising

Factorise $x^2 - 9x - 10$.

Find two numbers that multiply to make -10 and add to make -9.

These numbers are -10 and 1. Therefore $x^2 - 9x - 10 = (x - 10)(x + 1)$.

Factorise $6x^2 + x - 12$.

Find two numbers that multiply to make 6 × -12 = -72 and add to make 1. These two numbers are -8 and 9.

Therefore, $6x^2 + x - 12 = \underbrace{6x^2 - 8x} \underbrace{+ 9x - 12}$

$= 2x(3x - 4) + 3(3x - 4)$ (the two brackets must be identical)

$= (3x - 4)(2x + 3)$

Factorise expressions like $2x^2 + xy - 2x - y$ using the method of factorising by pairing.

$2x^2 + xy - 2x - y = x(2x + y) - 1(2x + y)$ (factorise front and back pairs, ensuring both brackets are identical)

$= (2x + y)(x - 1)$

---

4)

a) $x^2 - x - 6$

b) $x^2 + 6x - 16$

c) $2x^2 + 5x + 2$

d) $2x^2 - 3x$ (take out common factor)

e) $3x^2 + 5x - 2$

f) $2y^2 + 17y + 21$

g) $7y^2 - 10y + 3$

h) $10x^2 + 5x - 30$

i) $4x^2 - 25$

j) $x^2 - 3x - xy + 3y^2$

k) $4x^2 - 12x + 8$

l) $16m^2 - 81n^2$

m) $4y^3 - 9a^2y$

n) $8(x+1)^2 - 2(x+1) - 10$

Changing the Subject of a Formula

The formula $C = \frac{5(F-32)}{9}$ is used to convert between ° Fahrenheit and ° Celsius.
Rearrange to make F the subject.

$$C = \frac{5(F-32)}{9}$$

Multiply by 9	$9C = 5(F-32)$	(this removes the fraction)
Expand the brackets	$9C = 5F - 160$	
Add 160 to both sides	$9C + 160 = 5F$	
Divide both sides by 5	$\frac{9C+160}{5} = F$	$F = \frac{9C+160}{5}$

5) Make x the subject of each of these formulae:

a) $y = 7x - 1$

b) $y = \frac{x+5}{4}$

c) $4y = \frac{x}{3} - 2$

d) $y = \frac{4(3x-5)}{9}$

Make x the subject of $x^2 + y^2 = w^2$

Subtract y^2 from both sides:	$x^2 = w^2 - y^2$	(this isolates the term involving x)
Square root both sides:	$x = \pm\sqrt{w^2 - y^2}$	

Make a the subject of the formula $t = \frac{1}{4}\sqrt{\frac{5a}{h}}$

Multiply by 4	$4t = \sqrt{\frac{5a}{h}}$
Square both sides	$16t^2 = \frac{5a}{h}$
Multiply by h:	$16t^2 h = 5a$
Divide by 5:	$\frac{16t^2 h}{5} = a$

6) Make t the subject of each of the following

a) $P = \frac{wt}{32r}$

b) $P = \frac{wt^2}{32r}$

c) $V = \frac{1}{3}\pi t^2 h$

d) $P = \sqrt{\frac{2t}{g}}$

e) $Pa = \frac{w(v-t)}{g}$

f) $r = a + bt^2$

Solving Simultaneous Equations by Substitution

· Rearrange one equation to make one of the unknowns the subject

· Substitute the expression for this unknown into the other equation

This method may appear to be more complicated than elimination but it enables us to solve non - linear simultaneous equations.

<u>Example.</u>

Solve the simultaneous equations:

$$3r + 2s = 8 \text{ and } 2r - 4s = 16.$$

First name the equations to avoid confusion.

① $3r + 2s = 8$

② $2r - 4s = 16.$

Rearrange ① to make r the subject.

$$3r + 2s = 8 \qquad [-2s]$$

$$\Rightarrow 3r = 8 - 2s \qquad [\div 3]$$

$$\Rightarrow r = \frac{8-2s}{3}.$$

Substitute into ② . Use brackets to avoid mistakes.

$$2\left(\frac{8-2s}{3}\right) - 4s = 16. \cdots$$

Then solve, expanding the brackets carefully!

$$\frac{16-4s}{3} - 4s = 16 \qquad [+4s]$$

$$\Rightarrow \frac{16-4s}{3} = 16 + 4s \qquad [\times 3]$$

$$\Rightarrow 16 - 4s = 48 + 12s \qquad [+4s]$$

$$\Rightarrow 16 = 48 + 16s \qquad [-48]$$

$$\Rightarrow -32 = 16s \qquad [\div 16]$$

$$\Rightarrow -2 = s.$$

Then we substitute this value of s into one of the original equations to obtain the full solution,

$$3r - 4 = 8.$$

Hence $r = 4$.

7)

Solve the following pairs of simultaneous equations using the method of substitution.

a) $a + b = 7$
$2a + 3b = 18$

b) $s + 2t = 14$
$3s - t = 0$

c) $2a - 3b = 0$
$3a - 2b = 5$

d) $4w - 3t = 34$
$2t + 3w = 17$

e) $7p + 6 = 2r$
$3r - 2p = 26$

f) $z + 2l - 13 = 7$
$3z - 2l = -4$

Indices

y^4 means $y \times y \times y \times y$ 4 is called the **index** (plural: indices), **power** or **exponent** of y.

The 3 basic rules of indices:

$a^m \times a^n = a^{m+n}$ e.g. $3^4 \times 3^5 = 3^9$

$a^m \div a^n = a^{m-n}$ e.g. $3^8 \times 3^6 = 3^2$

$(a^m)^n = a^{mn}$ e.g. $(3^2)^5 = 3^{10}$

8) Simplify the following

a) $b \times 5b^5$ b) $3c^2 \times 2c^5$ c) $b^2c \times bc^3$

d) $2n^6 \times (-6n^2)$ e) $8n^8 \div 2n^3$ f) $d^{11} \div d^9$

g) $(a^3)^2$ h) $(-d^4)^3$

$a^0 = 1$ This result is true for any non-zero number a.

Therefore $5^0 = 1$ $\left(\frac{3}{4}\right)^0 = 1$ $(-5.2304)^0 = 1$

Fractional powers correspond to roots $a^{1/n} = \sqrt[n]{a}$

$8^{1/3} = \sqrt[3]{8} = 2$ $25^{1/2} = \sqrt{25} = 5$ $10000^{1/4} = \sqrt[4]{10000} = 10$ $4^{3/2} = \left(\sqrt{4}\right)^3 = 2^3 = 8$

9) Simplify

a) $4^{1/2}$ b) $27^{1/3}$ c) $\left(\frac{1}{9}\right)^{1/2}$

d) 5^{-2} e) 18^0 f) 7^{-1}

g) $27^{2/3}$ h) $\left(\frac{2}{3}\right)^{-2}$ i) $8^{-2/3}$

j) $\left(\frac{8}{27}\right)^{2/3}$ k) $\left(\frac{1}{16}\right)^{-3/2}$

Answers

1) a) $-15x + 21$ b) $-7a + 4$ c) $7x - 1$

2) a) $6x^2 + xy - 12y^2$ b) $4x^2 + 4x - 24$ c) $12 + 17x - 5x^2$

3) a) $9x^2 + 30x + 25$ b) $49x^2 - 28x + 4$ c) $9x^2 -1$ d) $25y^2 - 9$

4) a) $(x-3)(x+2)$ b) $(x+8)(x-2)$ c) $(2x+1)(x+2)$ d) $x(2x-3)$
e) $(3x -1)(x+2)$ f) $(2y+3)(y+7)$ g) $(7y-3)(y-1)$
h) $5(2x-3)(x+2)$ i) $(2x+5)(2x-5)$ j) $(x-3)(x-y)$
k) $4(x-2)(x-1)$ l) $(4m-9n)(4m+9n)$ m) $y(2y-3a)(2y+3a)$
n) $2(4x+5)(x-4)$

5) a) $x = \dfrac{y+1}{7}$ b) $x = 4y - 5$ c) $x = 3(4y + 2)$ d) $x = \dfrac{9y+20}{12}$

6) a) $t = \dfrac{32rP}{w}$ b) $t = \pm\sqrt{\dfrac{32rP}{w}}$ c) $t = \pm\sqrt{\dfrac{3V}{\pi h}}$
d) $t = \dfrac{P^2 g}{2}$ e) $t = v - \dfrac{Pag}{w}$ f) $t = \pm\sqrt{\dfrac{r-a}{b}}$

7) a) $a = 3, b = 4$ b) $s = 2, t = 6$ c) $a = 3, b = 2$
d) $w = 7, t = -2$ e) $p = 2, r = 10$ f) $z = 4, l = 8$

NOTES

LCHS AS Level Mathematics Induction Session